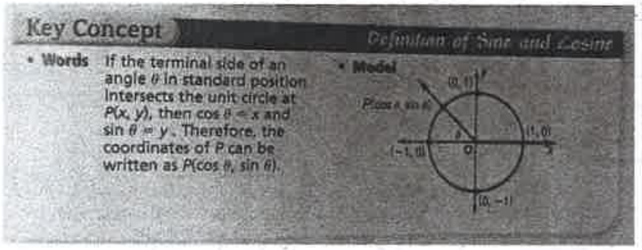
Unit Circle Definitions


Key Concept — Definition of Sine and Cosine

- **Words** If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos\theta = x$ and $\sin\theta = y$. Therefore, the coordinates of P can be written as $P(\cos\theta, \sin\theta)$.
- **Model**

Since there is exactly one point P(x,y) for an angle θ, the relations $\cos\theta = x$ *and* $\sin\theta = y$ are functions of θ. Because they are both defined using a unit circle, they are often called circular functions.

Example 1: Find sine and cosine given a point on the unit circle.

Given an angle θ in standard position, if $P\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$ lies on the terminal side and on the unit circle, find $\sin\theta$ and $\cos\theta$.

$\sin\theta = y = -\frac{1}{3}$

$\cos\theta = x = \frac{2\sqrt{2}}{3}$

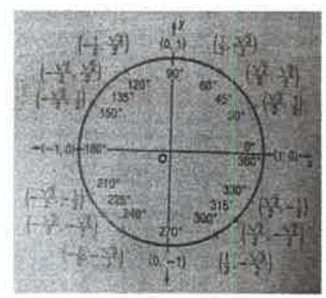
Example 2

Given an angle θ in standard position, if $P\left(\frac{3\sqrt{2}}{2}, \frac{3}{4}\right)$ lies on the terminal side and on the unit circle, find $\sin\theta$ and $\cos\theta$.

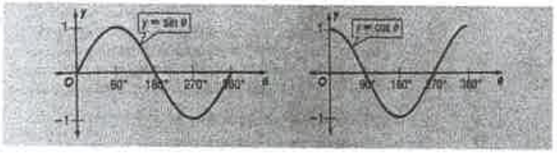
$\sin\theta = \frac{3}{4}$

$\cos\theta = \frac{3\sqrt{2}}{2}$

The exact values of the sine and cosine functions for specific angles are summarized using the definition of sine and cosine on the unit circle below:

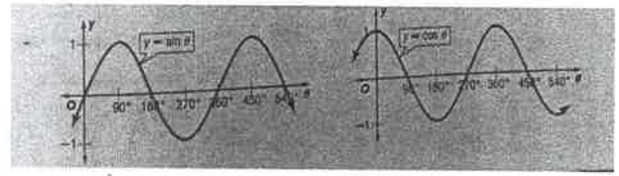


The same information is presented on the graphs of the sine and cosine functions below, where the horizontal axis shows the values of θ and the vertical axis shows the vales of $\sin\theta$ and $\cos\theta$.



Notice in the graph above that the values of sine for the conterminal angles 0° and 360° are both 0. The values of cosine for these are both 1. Every 360° or 2π radians, the sine and cosine functions repeat their values. So, we can say that the sine and cosine functions are <u>periodic</u>, each having a <u>period</u> of 360° or 2π radians

Periodic Function



A function is called periodic if there is a number a such that $f(x) = f(x + a)$ for all x in the domain of the function. The least positive value of a for which $f(x) = f(x + a)$ is called the period of the function.

Example 3: Find the exact value of each function.

- $\cos 675°$
- $\sin\left(-\frac{5\pi}{6}\right)$
- $\cos 690°$
- $\sin\left(-\frac{3\pi}{4}\right)$

$\cos 675 = \cos(315° + 360°)$

$\cos 315° = \boxed{\frac{\sqrt{2}}{2}}$

$\sin\left(-\frac{5\pi}{6}\right) = \sin\left(-\frac{5\pi}{6} + 2\pi\right)$

$= \sin\frac{7\pi}{6} = \boxed{-\frac{1}{2}}$

$\cos 690° = \cos(330 + 360)$

$= \cos 330 = \boxed{\frac{\sqrt{3}}{2}}$

$-\frac{3\pi}{4} = \sin\left(-\frac{3\pi}{4} + 2\pi\right)$

$= \sin\frac{5\pi}{4} = \boxed{\frac{-\sqrt{2}}{2}}$

Example 4:

As you ride a Ferris wheel, the height that you are above the ground varies periodically as a function of time. Consider the height of the center of the wheel to be the starting point. A particular wheel has a diameter of 38 feet and travels at a rate of 4 revolutions per minute.

a) Identify the period of this function.

4 ccw rotations/min.

$\frac{1}{4}$ of min. for 1 rotation $= \boxed{15 \text{ sec.}}$

b) Make a graph in which the horizontal axis represents the time t in seconds and the vertical axis represent the height h in feet in relation to the starting point.

d = 38

max & min = 19 (38 ÷ 2)