# Trig Functions of General Angles

## Something to Think About:

- How can you model the position of riders on a skycoaster?

On a certain skycoaster, after the first several swings, the angle of the riders from the center of the arch is given by:

$\theta = 0.2 \cos (1.6t)$

where t is the time in seconds after leaving the bottom of their swing.

## Remember When:

- Found values of trig functions whose domains were the set of all acute angles? For t > 0 in the previous equation, we must find the cosine of an angle greater than $\pi/2$

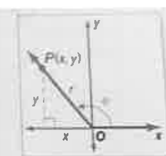
We will extend the domain of trigonometric functions to include angles of any measure.

## Trig Functions, $\theta$ in standard position

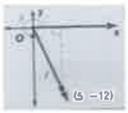
Let $\theta$ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of $\theta$. Using the Pythagorean Theorem, the distance $r$ from the origin to $P$ is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}, x \neq 0$

$\csc\theta = \frac{r}{y}, y \neq 0$ $\quad \sec\theta = \frac{r}{x}, x \neq 0$ $\quad \cot\theta = \frac{x}{y}, y \neq 0$



Example 1: Find the exact values of the six trigonometric functions of Θ if the terminal side of Θ contains the point (5,-12)




$x = 5$ use pythag. thm.
$y = -12$ to find r.

$r = \sqrt{x^2 + y^2}$

$\sqrt{5^2 + (-12)^2} = \sqrt{169} = 13$

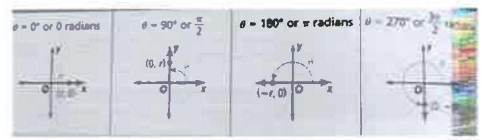
$\sin\theta = \frac{-12}{13}$ $\tan\theta = \frac{-12}{5}$ $\sec\theta = \frac{13}{5}$

$\cos\theta = \frac{5}{13}$ $\csc\theta = \frac{-13}{12}$ $\cot\theta = -\frac{5}{12}$

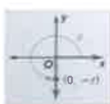
## Quadrantal Angles

- If the terminal side of angle Θ lies on one of the axes, Θ is called a **quadrantal angle**.
- 0, 90, 180, 360

*Since division by 0 is undefined, 2 trig values are undefined for each angle.




Example 2: Find the values of the six trig functions for an angle in standard position that measures 270°.




$x = 0$
$y = -r$

$\sin\theta = \frac{y}{r} = \frac{-r}{r} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0$

$\csc\theta = \frac{r}{y} = \frac{r}{-r} = -1$

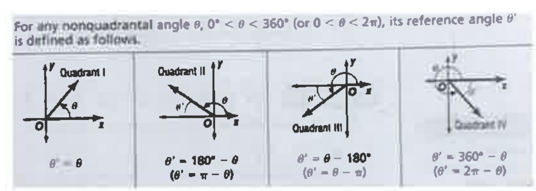
$\sec\theta = \frac{r}{x} = \frac{r}{0} =$ undefined

$\tan\theta = \frac{y}{x} = \frac{-r}{0}$ undefined

$\cot\theta = \frac{x}{y} = \frac{0}{-r} = 0$

## Reference Angles

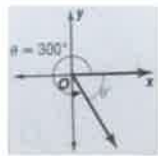
- If Θ is a nonquadrantal angle in standard position, its **reference angle** Θ′ is defined as the acute angle formed by the terminal side of Θ and the x-axis.



m Θ > 360 or < 0 its ref. < is found by associating it w/ a coterminal angle of positive measure between 0 & 360

## Example 3: Sketch each angle. Then find its reference angle.

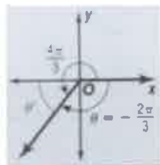
- 300°




$360 - 300 = 60°$
lies in Quad IV.

- $\frac{-2\pi}{3}$

$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$
lies in quad. 3
$\frac{4\pi}{3} - \pi = \frac{\pi}{3}$




## To use the reference angle Θ' to find a trig value of Θ:

- You need to know the sign of that function for an angle Θ.
  - From the function definitions these signs are determined by x and y since r is always positive.
  - Sign is determined by the quadrant.

| | Quadrant | | | |
|---|---|---|---|---|
| Function | I | II | III | IV |
| sin θ or csc θ | + | + | − | − |
| cos θ or sec θ | + | − | − | + |
| tan θ or cot θ | + | − | + | − |

## To find the value of a trig function of any angle Θ

1) Find the reference angle
2) Find the value of the trig function for the reference angle
3) Use the quadrant where the terminal side of Θ lies, determine the sign of the trig function value of Θ.

*Review 13-1 table of trig values for 30, 45, 60

## Example 4: Find the exact value of each trigonometric function.

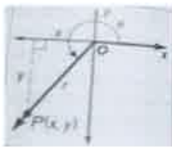
- Sin 120°


terminal side in QII
$180 - 120 = 60$
$\sin 120 = \sin 60 = \boxed{\frac{\sqrt{3}}{2}}$ + in QII

- Cot $\frac{7\pi}{4}$


term. side in QIV − in QIV
$2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$ $-\cot\frac{\pi}{4} = -\cot 45 = \boxed{-1}$

Example 5: Suppose Θ is an angle in standard position whose terminal side is in Quadrant III and sec Θ = $\frac{-4}{3}$. Find the exact values of the remaining five trig functions of Θ.



$\frac{r}{x} = \frac{-4}{3}$ $x = -3$ → based on location
$r = 4$

$(-3)^2 + y^2 = 4^2$
$y^2 = 16 - 9$
$y = \pm\sqrt{7}$

y is negative in QIII so
$y = -\sqrt{7}$

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{7}}{4}$

$\cos\theta = \frac{x}{r} = \frac{-3}{4}$

$\csc\theta = \frac{r}{y} = -\frac{4}{\sqrt{7}} = -\frac{4\sqrt{7}}{7}$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{7}}{-3} = \frac{\sqrt{7}}{3}$

$\sec\theta = \frac{-4}{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{-\sqrt{7}} = \frac{3\sqrt{7}}{7}$

Example 6: In a robotics competition, a robotic arm 4 meters long is to pick up an object at point A and release it into a container at point B. The robot's owner programs the arm to rotate through an angle of precisely 135° to accomplish this task. What is the new position of the object relative to the pivot point O?


A(4,0) $\theta' = 180 - 135 = 45°$

B(x,y) in Q2 so cos is neg., sin is pos.

$\cos 135 = \frac{x}{r}$ $\sin 135 = \frac{y}{r}$

$-\cos 45 = \frac{x}{4}$ $\sin 45 = \frac{y}{4}$

$-\frac{\sqrt{2}}{2} = \frac{x}{4}$ $\frac{\sqrt{2}}{2} = \frac{y}{4}$

$-2\sqrt{2} = x$ $2\sqrt{2} = y$

$B(-2\sqrt{2}, 2\sqrt{2})$

$2\sqrt{2} \approx 2.83$

2.83 m to the left and in front of the pivot point