9-6 Solving Rational Equations:

Solve each equation. Check for any extraneous solutions.

Extraneous solutions are any solutions that would make the denominator zero in the original problem. The problem would be undefined if that were the case so it is not a solution. ***Be sure to check all of your solutions!***

A) $\frac{7}{r+2}=\frac{6}{r-5}$

First method: cross multiply and solve **can only do this if you have one fraction equal to another**

$7(r-5) = 6(r+2)$

$7r-35 = 6r+12$

$r-35 = 12$

$\boxed{r=47}$

Second method: find common denominator and solve

$\frac{(r-5)}{(r-5)}\frac{7}{r+2} = \frac{6}{r-5}\frac{(r+2)}{(r+2)}$

$\frac{7(r-5)}{(r-5)(r+2)} = \frac{6(r+2)}{(r-5)(r+2)}$ → eliminate denominator once in common

$7(r-5) = 6(r+2)$

$7r-35 = 6r+12$

$\boxed{r=47}$

B) $\frac{x}{x+2}-\frac{x+2}{x-2}=\frac{x+3}{x-2}$

$\frac{x(x-2)}{(x+2)(x-2)} - \frac{(x+2)(x+2)}{(x+2)(x-2)} = \frac{(x+3)(x+2)}{(x+2)(x-2)}$

$(x^2-2x)-(x^2+4x+4) = x^2+5x+6$

$-6x-4 = x^2+5x+6$

$0 = x^2+11x+10 \rightarrow (x+10)(x+1)=0$

$\boxed{x=-10}$ $\boxed{x=-1}$

C) $\frac{x+1}{3x-6}=\frac{5x}{6}+\frac{1}{x-2}$

$\frac{x+1}{3(x-2)} = \frac{5x}{6}+\frac{1}{x-2}$

$\frac{2(x+1)}{6(x-2)} = \frac{5x(x-2)}{6(x-2)}+\frac{6}{6(x-2)}$

$2x+2 = 5x^2-10x+6$

$5x^2-12x+4=0$

$(5x-2)(x-2)$

$5x-2=0$

$\boxed{x=2/5}$

$x-2=0$

$x=2$

↓ makes denom = 0 so not a solution

D) $\frac{r+2}{2r+1}=\frac{r}{3}+\frac{3}{4r+2}$

$\frac{r+2}{2r+1} = \frac{r}{3}+\frac{3}{2(2r+1)}$

$\frac{6(r+2)}{6(2r+1)} = \frac{2r(2r+1)}{6(2r+1)}+\frac{9}{6(2r+1)}$

$6r+12 = 4r^2+2r+9$

$4r^2-4r-3=0$

$2r+1=0$ $\quad$ $2r-3=0$

$r=1/2$ X $\quad$ $\boxed{r=3/2}$ ✓

E) $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

$\frac{12(4)}{4x}+\frac{3x}{4x} = \frac{6x}{4x}$

$48+3x=6x$

$48=3x$

$\boxed{16=x}$

F) $\frac{6}{x-1}=\frac{4}{x-2}+\frac{2}{x-1}$

$$\frac{6(x-2)}{(x-1)(x-2)}=\frac{4(x-1)}{(x-2)(x-1)}+\frac{2(x-2)}{(x-1)(x-2)}$$

$6x-12 = 4x-4+2x-4$

$6x-12 = 6x-8$

$-12 = -8$

no solution

G) $\frac{2}{1}=\frac{x+2}{x-3}+\frac{x-2}{x-6}$

$$\frac{2(x-3)(x-6)}{(x-3)(x-6)}=\frac{(x+2)(x-6)}{(x-3)(x-6)}+\frac{(x-2)(x-3)}{(x-6)(x-3)}$$

$2x^2-18x+36 = x^2-4x-12+x^2-5x+6$

$2x^2-18x+36 = 2x^2-9x-6$

$-18x+36 = -9x-6$

$-9x+36 = -6$

$-9x = -42$

$x = 42/9 = \boxed{14/3}$

H) $\frac{x^2+4}{x^2-4}+\frac{x}{2-x}=\frac{2}{x+2}$

$$\frac{x^2+4}{(x+2)(x-2)}+\frac{x}{2-x}=\frac{2}{x+2}$$

* $2-x$ is opposite of $x-2$ so multiply by $\frac{-1}{-1}$.

$$\frac{x^2+4}{(x+2)(x-2)}-\frac{x}{x-2}=\frac{2}{x+2}$$

$$\frac{x^2+4}{(x+2)(x-2)}-\frac{x(x+2)}{(x-2)(x+2)}=\frac{2(x-2)}{(x+2)(x-2)}$$

$x^2+4-(x^2+2x) = 2x-4$

$4-2x = 2x-4$

$4 = 4x-4$

$8 = 4x$

$x = 2$ X

no solution

↳ makes denom. = 0