Algebra 2, Unit 1 Name Key

Review: Sequences and Series

Date

For each sequence, state if it is arithmetic, geometric, or neither. State the pattern, and find the next 3 terms.

1) 10, 12, 15, 19, 24, ... (+2, +3, +4, +5) 30, (+6) 37, (+7) 45, (+8)

neither

2) 14, 44, 74, 104, 134, ... 164, 194, 224

arithmetic

$d = 30$

3) 3, −9, 27, −81, 243, ... −729, 2187, −6561

geometric

$r = -3$

4) −2, −4, −12, −48, −240, ... (×2, ×3, ×4, ×5, ×6) −1440, (×7) −1080, (×8) −80,640

neither

Find the common difference and the term named in the problem.

5) 34, 54, 74, 94, ... $a_{34} = 34 + 20(34-1)$

Find a_{34}

$d = 20$

$a_{34} = 694$

6) 21, 12, 3, −6, ... $a_{36} = 21 + -9(36-1)$

Find a_{36}

$d = -9$

$a_{36} = -294$

Given the first term and the common difference of an arithmetic sequence find the term named in the problem and the explicit formula.

7) $a_1 = -34,\ d = -20$

Find a_{34}

$a_n = -34 - 20(n-1)$

$a_n = -34 - 20n + 20$

$a_n = -14 - 20n \rightarrow$ explicit formula

$a_{34} = -694$

8) $a_1 = 22,\ d = 8$

Find a_{34}

$a_n = a_1 + d(n-1)$

$a_n = 22 + 8(n-1)$

$a_n = 22 + 8n - 8$

$a_n = 14 + 8n \rightarrow$ explicit formula

$a_{34} = 286$

Find the missing term or terms in each arithmetic sequence.

9) ..., 26, 33, 40, 47, 54, ... (a_1, a_5)

Find common difference

$54 = 26 + d(5-1)$

$54 = 26 + 4d$

$28 = 4d$

$\boxed{d = 7}$

Evaluate each arithmetic series described.

10) $a_1 = 10,\ a_n = 105,\ n = 20$

$S_n = n/2\,(a_1 + a_n)$

$S_{20} = 20/2\,(10 + 105)$

$10(115) = \boxed{1150}$

Express the arithmetic series in Sigma Notation. Find the sum of the finite series.

11) $11.9 + 13.1 + 14.3 + 15.5\ldots,\ n = 15$

$\sum_{n=1}^{15} 1.2n + 10.7$

$a_n = a_1 + d(n-1)$

$11.9 + 1.2(n-1)$

$11.9 + 1.2n - 1.2$

$10.7 + 1.2n$

$a_1 = 11.9\quad a_{15} = 28.7$

$S_{15} = 15/2\,(11.9 + 28.7) = \boxed{304.5}$

12) $10 + 20 + 30 + 40\ldots,\ n = 16$

$\sum_{n=1}^{16} 10n$

$a_1 = 10$

$a_{16} = 160$

$a_n = 10 + 10(n-1)$

$10 + 10n - 10$

$a_n = 10n$

$S_{16} = 16/2\,(10 + 160)$

$= \boxed{1360}$

Evaluate each arithmetic series described.

13) $\sum_{i=1}^{5} (10i - 9)$

$a_1 = 10(1) - 9 = 1$

$a_5 = 10(5) - 9 = 41$

$S_5 = 5/2\,(1 + 41) = \boxed{105}$

14) $\sum_{m=4}^{48} (8m + 2)$

* pay attention to where the series starts from!

$a_4 = 8(4) + 2 = 34$

$a_{48} = 8(48) + 2 = 386$

total terms $= 48 - 4 + 1 = 45$

$S_{45} = 45/2\,(34 + 386)$

$\boxed{S_{45} = 9450}$

Find the common ratio and the term named in the problem.

15) 4, 12, 36, 108, ...

Find a_{10}

$a_n = a_1 \cdot r^{n-1}$

$r = 3$

$a_{10} = 4 \cdot 3^9 = \boxed{78,732}$

16) –2, 6, –18, 54, ...

Find a_{11}

$r = -3$

$a_{11} = -2 \cdot -3^{10}$

$\boxed{a_{11} = -118,098}$

Given the first term and the common ratio of a geometric sequence find the term named in the problem and the explicit formula.

17) $a_1 = 3,\ r = 4$

Find a_9

$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot 4^{n-1}$ → explicit formula

$a_9 = 3 \cdot 4^8$

$= \boxed{196,608}$

Find the term named in the problem and the explicit formula.

18) $-1, -3, -9, -27, \ldots$ Find a_9

$a_1 = -1$, $r = 3$

$a_n = -1 \cdot 3^{n-1}$ → explicit formula

$a_9 = -1 \cdot 3^8 = \boxed{-6561}$

Find the missing term or terms in each geometric sequence.

19) $\ldots, -1, -3, -9, -27, -81, \ldots$ (×3, ×3, ×3)

find common ratio.

$a_1 = -1$

$a_5 = -81$

$n = 5$

$a_5 = a_1 \cdot r^{5-1}$

$-81 = -1 \cdot r^4$

$81 = r^4$

$r = 3$

Evaluate each geometric series described.

20) $a_1 = -4$, $a_8 = -312500$, $r = 5$ $\quad S_n = \dfrac{a_1 - a_1 r^n}{1-r}$

$S_8 = \dfrac{-4 - -4(5)^8}{1-5}$

$\boxed{S_8 = -390{,}624}$

21) $a_1 = -4$, $r = -3$, $n = 10$

$S_{10} = \dfrac{-4 - -4(-3)^{10}}{1--3}$

$\boxed{S_{10} = 59{,}048}$

22) $\sum_{i=1}^{7} 4 \cdot (-4)^{i-1}$ $\quad a_1 = 4$ $\quad r = -4$ $\quad n = 7$

$S_7 = \dfrac{4 - 4(-4)^7}{1--4} = \boxed{13{,}108}$

23) $\sum_{k=1}^{10} (-3)^{k-1}$ $\quad a_1 = 1$ $\quad r = -3$ $\quad n = 10$

$S_{10} = \dfrac{1 - 1(-3)^{10}}{1--3} = \boxed{-14762}$

* watch your signs with this one!

Express each geometric series in sigma notation. What is the sum of the finite series?

24) $4 + 16 + 64 + 256\ldots, n = 6$ $\quad a_n = a_1 \cdot r^{n-1}$

$\sum_{n=1}^{6} 4 \cdot 4^{n-1}$

$a_1 = 4$

$r = 4$

$n = 6$

$a_n = 4 \cdot 4^{n-1}$

$S_6 = \dfrac{4 - 4(4)^6}{1-4}$

$S_6 = \boxed{5460}$

25) $4 - 8 + 16 - 32\ldots, n = 8$

$\sum_{n=1}^{8} 4(-2)^{n-1}$

$a_1 = 4$

$r = -2$

$n = 8$

$a_n = 4 \cdot -2^{n-1}$

$S_8 = \dfrac{4 - 4(-2)^8}{1--2} = \boxed{-340}$