Pythagorean Theorem Notes

In any **right** triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. This is known as the Pythagorean theorem and is stated as:

$$a^2 + b^2 = c^2$$

a and b are legs

c is the hypotenuse and is found across from the right angle

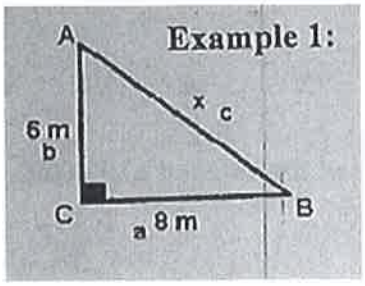
There are certain sets of numbers that not only satisfy the Pythagorean theorem, but any multiples of these numbers will also satisfy the Pythagorean theorem.

The special sets of numbers that have this property are called Pythagorean triples.

Example: 3, 4, and 5 satisfy the Pythagorean Theorem. If you multiply all three numbers by 2 (6, 8, 10), these new numbers also satisfy the Pythagorean Theorem.

Pythagorean Triples MUST be whole numbers. The most common sets of Pythagorean triples are:

1) 3, 4, 5
2) 5, 12, 13
3) 8, 15, 17



Find x. $= 10$

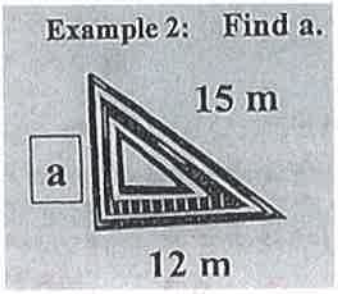
$a^2 + b^2 = c^2$

$8^2 + 6^2 = c^2$

$64 + 36 = c^2$

$c^2 = 100$

$c = 10$



$a^2 + 12^2 = 15^2$

$a^2 + 144 = 225$

$a^2 = 81$

$a = 9$

Example 3: A triangle has sides of 6, 7, and 10. Is it a right triangle?

↳ longest, so must be c.

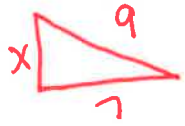
$10^2 \stackrel{?}{=} 6^2 + 7^2$

$100 = 36 + 49$

$100 \neq 85$

not a right triangle

Example 4: A ramp was constructed to load a truck. If the ramp is 9 feet long and the horizontal distance from the bottom of the ramp to the truck is 7 feet, what is the vertical height of the ramp?



$a^2+b^2=c^2$

$x^2+7^2=9^2$

$x^2=32$

$x=\sqrt{32}$

$x=5.66$ ft

Example 5: Determine whether $\triangle ABC$ is a right triangle for the given vertices.

A(-9,-3) B(1,-1) C(-3,-7)

$AB=\sqrt{(-9-1)^2+(-3+1)^2}=\sqrt{100+4}=\sqrt{104}$

$BC=\sqrt{(1+3)^2+(-1+7)^2}=\sqrt{16+36}=\sqrt{52}$

$AC=\sqrt{(-9+3)^2+(-3+7)^2}=\sqrt{36+16}=\sqrt{52}$

$(\sqrt{52})^2+(\sqrt{52})^2=(\sqrt{104})^2$ $\quad 52+52=104$ ✓ right $\triangle$

Example 6: Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.

A) 9, 12, 15

B) 1, 2, 3

C) $\frac{\sqrt{3}}{6}, \frac{\sqrt{6}}{5}, \frac{3}{5}$

$9^2+12^2=15^2$ yes, right $\triangle$ yes Pyth. trip.

not right $\triangle$

$\left(\frac{\sqrt{3}}{5}\right)^2+\left(\frac{\sqrt{6}}{5}\right)^2=\left(\frac{3}{5}\right)^2$

$9/25=9/25$ yes right $\triangle$ no pyth. trip.

$Hyp^2 < leg^2 + leg^2 \Rightarrow$ ACUTE
$Hyp^2 = leg^2 + leg^2 \Rightarrow$ RIGHT
$Hyp^2 > leg^2 + leg^2 \Rightarrow$ OBTUSE

Example 7: Determine whether each set of numbers can be measures of the sides of a triangle. If so, classify the triangle as acute, obtuse, or right. Justify your answer.

A) 9, 12, 15

$15^2=9^2+12^2$

$225=225$ right

B) 2, 5, 10

$2+5<10$ not a $\triangle$

C) 9, 9, 13

$13^2=9^2+9^2$

$169>162$ obtuse

D) 8, 10, 12

$12^2=8^2+10^2$

$144<164$ acute

* If $\triangle$: 2 sides add to be more than 3rd