Chapter 7 Review: Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $2n^2 + 2m^2$ — not a polynomial in one variable

2. $5 - 3a^2$ — degree: 2, leading coeff: -3

3. $(x^2 + 2)(x^3 - 5)$ — $x^5 - 5x^2 + 2x^3 - 10$; degree: 5, lead. coeff: 1

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

4. $(x^3 - x^2 + x + 14); (x + 2)$

$-2 \mid 1 \quad -1 \quad 1 \quad 14$
$\quad\quad\downarrow \quad -2 \quad 6 \quad -14$
$\quad\quad\quad 1 \quad -3 \quad 7 \quad \underline{0}$

factors: $(x^2 - 3x + 7)(x + 2)$

6. $(x^3 - 8); (x - 2)$

$2 \mid 1 \quad 0 \quad 0 \quad -8$
$\quad\downarrow \quad 2 \quad 4 \quad 8$
$\quad\quad 1 \quad 2 \quad 4 \quad \underline{0}$

factors: $(x^2 + 2x + 4)$
$(x - 2)$

5. $(2x^3 + x^2 - 41x + 20); (x - 4)$

$4 \mid 2 \quad 1 \quad -41 \quad 20$
$\quad\downarrow \quad 8 \quad 36 \quad -20$
$\quad\quad 2 \quad 9 \quad -5 \quad \underline{0}$

$2x^2 + 9x - 5 = 0$

$x^2 + 9x - 10$

$(x + 10)(x - 1)$
$\quad\quad 2 \quad\quad 2$

factors: $(x + 5)(2x - 1)(x - 4)$

Solve each equation and state the number and type of roots.

7. $-5x - 7 = 0$ — 1 zero, real

$-5x = 7$

$x = -7/5$

8. $3x^2 + 10 = 0$ — 2 zeros, complex

$3x^2 = -10$

$x^2 = -\frac{10}{3}$

$x = \pm\sqrt{-\frac{10}{3}} = \pm i\frac{\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm i\frac{\sqrt{30}}{3}$

9. $(x^4 - 2x^3 - 7x^2 - 2x - 8 = 0$

$-2, 4, \pm i$

use synthetic division, then solve the quad.

2 real, 2 imag

10. $x^4 - 2x^3 = 23x^2 - 60x$

$-5, 0, 3, 4$

4 real

Find all zeros of the function.

11. $f(x) = x^3 - 7x^2 + 16x - 10$

1	1	-7	16	-10
	↓	1	-6	10
	1	-6	10	0

$x^2 - 6x + 10 = 0$

$\frac{6 \pm \sqrt{-6^2 - 4(1)(10)}}{2(1)} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$

$x = 1$

$x = 3 \pm i$

12. $f(x) = 10x^3 + 7x^2 - 82x + 56$

2	10	7	-82	56
	↓	20	54	-56
	10	27	-28	0

$10x^2 + 27x - 28$

$x = \frac{27 \pm \sqrt{27^2 - 4(10)(-28)}}{2(10)} = \frac{27 \pm \sqrt{1849}}{20}$

$\frac{27 \pm 43}{20}$

$\frac{70}{20} = 3.5$

$\frac{-16}{20} = -\frac{4}{5}$

13. $f(x) = x^3 - 16x^2 + 79x - 114$

i	1	-16	79	-114
	↓	i	-10i	11i + 114
	1	-16 + i	79 - 10i	0

$x^2 - 10x + 19$

$x = \frac{10 \pm \sqrt{-10^2 - 4(1)(19)}}{2(1)} = \frac{10 \pm \sqrt{24}}{2} = 5 \pm \sqrt{6}$

Write a polynomial function of least degree with the integral coefficients that has the given zeros.

14. -3, 1, 2

$x = -3 \quad x = 1 \quad x = 2$

$(x+3)(x-1)(x-2)$

$(x^2 + 2x - 3)(x - 2) = x^3 - 7x + 6$

15. -6, 6, -5i, 5i

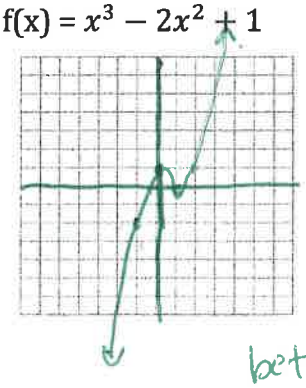
$(x+6)(x-6)(x+5i)(x-5i)$

$(x^2 - 36)(x^2 + 25)$

$x^4 - 11x^2 - 900$

Graph each function by making a table of values. Determine the values of x at which or between which each real zero is located.

16. $f(x) = x^3 - 2x^2 + 1$



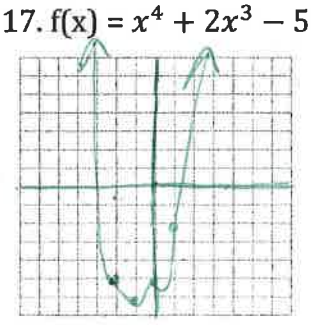
x	y
-2	-15
-1	-2
0	1
1	0
2	1

between -1 & 0

at 1

between 1 & 2

17. $f(x) = x^4 + 2x^3 - 5$



x	y
-2	-5
-1	-6
0	-5
1	-2
2	27

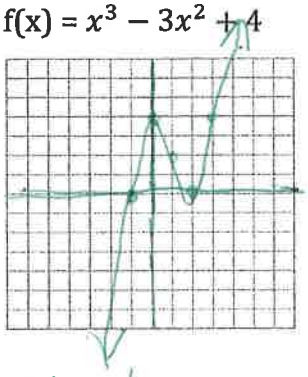
between -3 & -2

between 1 & 2

18. $f(x) = -x^4 + 2x^2 - 1$

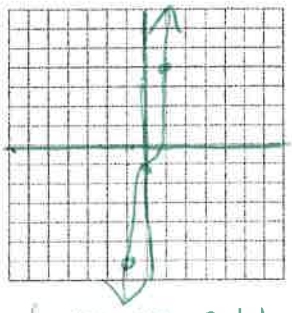
at 1 and
at −1

19. $f(x) = x^3 - 3x^2 + 4$



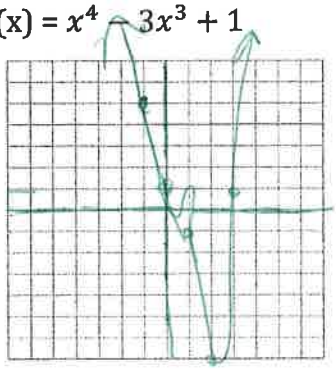
at −1
and at 2

20. $f(x) = 3x^3 + 2x - 1$



between 0 & 1

21. $f(x) = x^4 - 3x^3 + 1$

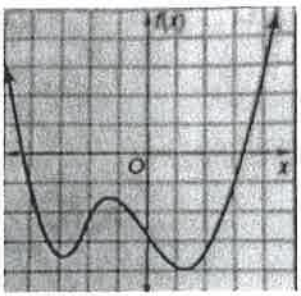


between 0 & 1
between 2 & 3

For each graph,

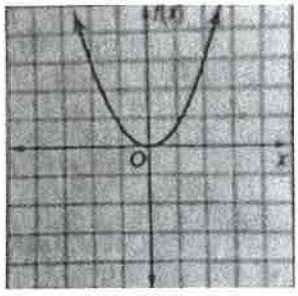
a. describe the end behavior
b. determine whether it represents an odd-degree or an even-degree polynomial function and
c. state the number of real zeroes

22.



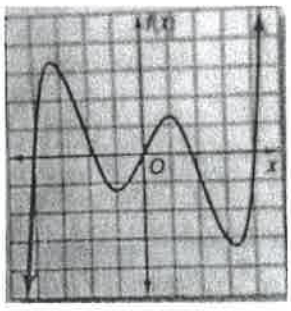
a) up, up
b) even
c) 2

23.



a) up up
b) even
c) 1

24.



a) down up
b) odd
c) 5