Permutations Notes

When a group of objects or people are arranged in a particular order, the arrangement is called a **_permutation._**

In permutations, ORDER MATTERS!

The **factorial (!)** can be used to make things much faster when multiplying. Start with the number preceding the factorial or exclamation point and multiply by each lower number all the way to one.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$

$4! = 4 \times 3 \times 2 \times 1 = 24$

To find factorial (!) on the calculator: Math ➜ PRB ➜ 4

The formula for permutations if "n" elements are taken "r" at a time:

$$nPr \text{ or } \frac{n!}{(n-r)!}$$

To do permutations on the calculator: Enter the first number followed by Math ➜ PRB ➜ 2 (nPr) then the last number.

Find 8 P 3: 336 Find 7 P 2: 42 Find 10 P 8: $1,814,400$

Examples:

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$\overline{}$
$5 \times 4 \times 3 \times 2 \times 1$

A) How many answer sheets are possible for a 5 question true/false quiz? $\underline{2} \ \ \underline{2} \ \ \underline{2} \ \ \underline{2} \ \ \underline{2} = 32$

B) How many answer sheets are possible on a quiz with 3 true/false questions and 3 multiple choice questions with choices A-D?
$\underline{2} \ \ \underline{2} \ \ \underline{2} \ \ \underline{4} \ \ \underline{4} \ \ \underline{4} = 512$

C) Find the number of permutations of {A, B, C, D, E, F}
   Note: when letters appear in brackets with permutations, we can assume they do not repeat.
   $6! = 720$
   $6 P 6$

D) Find the number of permutations of {M A T H}

$$4! = 24$$

E) There are four people on a list. How many different ways can 1st and second place be selected?

$$4P2 = 12$$

F) How many different permutations of {M A T H E M A T I C S} exist?
For this type of problem, the factorial of the total number of letters is taken over the factorial of each repeated letter.

$$\frac{11! \text{ total}}{2! \, 2! \, 2! \text{ each repeated letter}} = 4,989,600$$

G) How many different permutations of {SUCCEEDED} exist?

$$\frac{9!}{2! \, 3! \, 2!} = 15,120$$

H) A 10 question quiz contains 3 true/false, 2 multiple choice (choose A, B, or C) and 5 multiple choice (choose A, B, C, D). If you know that questions 4, 7 9, and 10 are correct, how many possible answer sheets will there be?

$$\underline{2} \ \underline{2} \ \underline{2} \ \underline{1}_{4} \ \underline{3} \ \underline{4} \ \underline{1}_{7} \ \underline{4} \ \underline{1}_{9} \ \underline{1}_{10} = 384$$

I) The coach is selecting a 5 position baseball line up to bat from 10 players. How many arrangements are possible if the best player is up second?

$$\underline{9} \ \underline{1} \ \underline{8} \ \underline{1} \ \underline{6} = 3024$$

J) How many possible phone numbers are there if the first number is not 0 and the last number is an even number, but not 4?

$$\underline{9} \ \underline{10} \ \underline{10} \quad \underline{10} \ \underline{10} \ \underline{10} \ \underline{4} = 3,600,000$$

K) A 3-letter code word starts with a vowel and ends with a consonant. How many code words are possible?

$$\underline{5} \ \underline{26} \ \underline{21} = 2730$$

L) How many different ways can 4 people stand in a line?

$$4! = 24$$

M) If there is a 3-digit number and repetition is not allowed, how many numbers are possible?

$$\underline{9} \ \underline{9} \ \underline{8} = 648$$