

Properties of Logarithms

Solve each equation. Check your answer.

1)  $3 \log_7 4 = 2 \log_7 b$

$$4^3 = b^2$$

$$64 = b^2$$

$$8 = b$$

2)  $\log_6 2c + \log_6 8 = \log_6 80$

$$16c = 80$$

$$c = 5$$

3)  $\log_2 q - \log_2 3 = \log_2 7$

$$\frac{q}{3} = 7$$

$$q = 21$$

4)  $3 \log_8 2 - \log_8 4 = \log_8 b$

$$\frac{2^3}{4} = b$$

$$\frac{8}{4} = b$$

$$2 = b$$

5)  $\log_4 x + \log_4 (2x - 3) = \log_4 2$

$$x(2x - 3) = 2$$

$$2x^2 - 3x = 2$$

$$2x^2 - 3x - 2 = 0$$

$$(2x + 1)(x - 2) = 0$$

$$x = -\frac{1}{2} \quad x = 2$$

6)  $\log_2 (x + 4) - \log_2 (x - 3) = 3$

$$\log_2 \frac{x+4}{x-3} = 3$$

$$\frac{2^3}{1} = \frac{x+4}{x-3}$$

$$8x - 24 = x + 4$$

$$x = 4$$

Base e and Natural Logarithms

Write an equivalent exponential or logarithmic equation.

7)  $\ln 50 = x$

$$e^x = 50$$

8)  $\ln 36 = 2x$

$$e^{2x} = 36$$

9)  $e^x = 8$

$$\ln 8 = x$$

10)  $e^5 = 10x$

$$\ln 10x = 5$$

Solve each equation or inequality. Round to four decimal places.

11)  $e^x < 9$

$$x < \ln 9$$

$$x < 2.1972$$

12)  $e^{-x} = 31$

$$-x = \ln 31$$

$$x = -3.4340$$

13)  $2e^x - 3 = 1$

$$2e^x = 4$$

$$e^x = 2$$

$$x = \ln 2$$

$$x = 0.6931$$

14)  $5e^x + 1 \geq 7$

$$5e^x \geq 6$$

$$e^x \geq \frac{6}{5}$$

$$x \geq .1827$$

15)  $e^{3x} = 8$

$$3x = \ln 8$$

$$x = .6931$$

16)  $e^{-4x} = 5$

$$-4x = \ln 5$$

$$x = -.4024$$

17)  $e^{2x} + 1 = 55$

$$e^{2x} = 54$$

$$2x = \ln 54$$

$$x = 1.9945$$

18)  $e^{3x} - 5 = 32$

$$e^{3x} = 37$$

$$3x = \ln 37$$

$$x = 1.2036$$

19)  $\ln 4x = 3$

$$4x = e^3$$

$$x = 5.0214$$

20)  $\ln(-2x) = 7$

$$-2x = e^7$$

$$x = -548.3166$$

21)  $\ln(x + 2) = 3$

$$x + 2 = e^3$$

$$x = 18.0855$$

22)  $\ln(x + 3) = 5$

$$x + 3 = e^5$$

$$x = 145.413$$

Application

- 23) Felipe is saving money to take a vacation. He invests \$800 at 6% interest compounded monthly. How many years will it take for his investment to reach \$1500?

10.5 years

- 24) William deposited \$2200 at 8% interest compounded semiannually. When will the account be worth \$3810?

7 years

- 25) A fossil now contains 75 mg of Carbon 14. There were originally 125 mg of Carbon 14 in the fossil. If the half-life of carbon is 5570 years, find the approximate age of the fossil to the nearest year.

4105 years

- 26) Approximate the age of a bone that now contains 84 g of carbon 14 if it originally contained 192 g of that isotope. The half-life of carbon 14 is 5570 years. Round the answer to the nearest 100 year.

6600 years

- 27) If the voltage output of an amplifier is 36 volts and its voltage input is 0.9 volts, what is the decibel voltage gain?

16.02 decibels

- 28) What is the decibel voltage gain of an amplifier with an output voltage of 50 volts and an input voltage of 0.6 volts?

19.21 decibels

Properties of Logarithms

Solve each equation. Check your answer.

1)  $\log_3(x+2) + \log_3 6 = 3$

$$\log_3(6x+12) = 3$$

$$27 = 6x + 12$$

$$15 = 6x$$

$$\boxed{2.5 = x}$$

2)  $\log_6 x - 2 \log_6 2 = 3 \log_6 3$

$$\frac{x}{2^2} = \frac{3^3}{1}$$

$$\frac{x}{4} = \frac{27}{1}$$

$$\boxed{x = 108}$$

3)  $\log_2 5 + (1/3) \log_2 27 = \log_2 x$

$$5(27^{1/3}) = x$$

$$5(3) = x$$

$$\boxed{15 = x}$$

4)  $\log_8 42 - \log_8 x = \log_8 7$

$$\frac{42}{x} = \frac{7}{1}$$

$$7x = 42$$

$$\boxed{x = 6}$$

5)  $\log_4 x + \log_4 9 = \log_4 45$

$$9x = 45$$

$$\boxed{x = 5}$$

Exponential and Natural Logarithms

Write an equivalent exponential or logarithmic equation.

6)  $\ln 20 = x$

$$\boxed{e^x = 20}$$

7)  $e^x = 15$

$$\boxed{\ln 15 = x}$$

Solve each equation or inequality. Round to four decimal places.

8)  $e^x < 5$

$$x < \ln 5$$

$$\boxed{x < 1.6094}$$

9)  $e^{-2x} = 4$

$$-2x = \ln 4$$

$$\boxed{x = -0.6931}$$

10)  $2e^x - 7 = 9$

$$2e^x = 16$$

$$e^x = 8$$

$$x = \ln 8$$

$$\boxed{x = 2.0794}$$

11)  $3e^x + 2 \geq 11$

$$3e^x \geq 9$$

$$e^x \geq 3$$

$$x \geq \ln 3$$

$$\boxed{x \geq 1.0986}$$

12)  $\ln 2x = 7$

$$2x = e^7$$

$$\boxed{x = 548.3166}$$

13)  $\ln(-3x) = 8$

$$-3x = e^8$$

$$\boxed{x = -993.6527}$$

14)  $\ln(x+4) = 5$

~~$$x+4 = e^5$$~~

$$x+4 = e^5$$

$$\boxed{x = 144.4132}$$

15)  $\ln(x+3) = 5$

$$x+3 = e^5$$

$$\boxed{x = 145.4132}$$

## Application

- 16) Phillip is saving money to take a vacation. He invests \$900 at 7% interest compounded quarterly. How many years will it take for his investment to reach \$2500?

$$2500 = 900 \left(1 + \frac{0.07}{4}\right)^{4t}$$

$$\frac{25}{9} = 1.0175^{4t}$$

$$\log\left(\frac{25}{9}\right) = 4t \log 1.0175$$

$$58.889 = 4t$$

$$t \approx 14.7 \text{ yrs}$$

- 17) James invests \$4000 at 6.5% annual interest, compounded monthly. How long will it take to this money to double in value?

$$8000 = 4000 \left(1 + \frac{0.065}{12}\right)^{12t}$$

$$2 = 1.0054^{12t}$$

$$\log 2 = 12t \log 1.0054$$

$$128.707 = 12t$$

$$t = 10.7 \text{ yrs}$$

- 18) If \$3000 is deposited at an annual rate of 5% compounded continuously, how long will it take for the account to reach \$6000? (you may use  $A = Pe^{rt}$ )

$$6000 = 3000 e^{.05t}$$

$$2 = e^{.05t}$$

$$\ln 2 = .05t$$

$$13.9 \text{ yrs} = t$$

- 19) If the voltage output to an amplifier is 40 volts and its voltage input is 0.8 volts, what is the decibel voltage gain?

$$10 \log\left(\frac{40}{0.8}\right) = 16.99 \text{ volts}$$

- 20) A skull if found at an archaeological site contains 200 mg of Carbon 14. The sample originally contained 600 mg of that isotope. If the half life for Carbon 14 is 5570 years, find the approximate age of the bone in the nearest 100 years.

$$A = A_0 2^{-t/k}$$

$$200 = 600 (2)^{-t/5570}$$

$$\log\left(\frac{1}{3}\right) = -t/5570 \log 2$$

$$-1.585 = -t/5570$$

$$t = 8828 \text{ yrs}$$

$$\approx 8800 \text{ yrs}$$