Notes - . .pplication Using Logs

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\quad A = Pe^{rt}$

Compound Interest

A) Rita puts $3000 in a bank at 8% annual interest, compounded semiannually. How long will it take the account to earn $1800 interest?

$4800 = 3000\left(1 + \frac{0.08}{2}\right)^{2t}$ ≈ 6 years

$1.6 = 1.04^{2t}$

$\log 1.6 = 2t \log 1.04$ $\quad 11.98 = 2t$

$\frac{\log 1.6}{\log 1.04} = 2t$ $\quad 5.99 = t$

B) Kyle deposited $1400 for 8 years at 6½% interest compounded quarterly. How much will he have at the end of 8 years? How much interest has he earned?

$A = 1400\left(1 + \frac{.065}{4}\right)^{4(8)}$

$A = 1400(1.01625)^{32}$

$A =$ $2345 total after 8 yrs.

$2345 - 1400 =$ $945 Interest in 8 yrs

C) How long will a principal of $5000 at 9% interest compounded semiannually take to reach $8479 if that is the cost of the used car that Bob has bought?

$8479 = 5000\left(1 + \frac{0.09}{2}\right)^{2t}$

$\frac{8479}{5000} = 1.045^{2t}$

$\log\left(\frac{8479}{5000}\right) = 2t \log 1.045$

$11.99 = 2t$

$5.99 = t$

≈ 6 years

D) Jane is saving money for a trip at high school graduation 5 years from now. If a 5 year certificate of deposit pays 7.25% interest compounded continuously, how much should she invest to have $3000 for the trip? Use $A = Pe^{rt}$ and round to the nearest dollar.

$3000 = Pe^{(0.0725)(5)}$

$3000 = Pe^{0.3625}$

$2088 = P$

$2088

E) Jake deposited $200 in an account at 5% interest, compounded continuously. How long will it take him to double his money? Use Pe^{rt} and round your answer to the nearest year.

$400 = 200e^{0.05t}$

$2 = e^{0.05t}$

$\ln 2 = 0.05t$

$13.86 = t$

13.86 years

14 years

Decibel Voltage Gain

$$10 \log \frac{E_o}{E_I}$$

F) If input to an amplifier is 0.6 volts and output is 40 volts, find the decibel voltage gain.

$$10 \log\left(\frac{40}{0.6}\right)$$

$$= 18.24 \text{ decibels}$$

G) An amplifier delivers a 37 decibel voltage gain. If the input voltage is 0.9, find the output voltage.

$$37 = 10 \cdot \log\left(\frac{E_o}{0.9}\right)$$

$$3.7 = \log \frac{E_o}{0.9}$$

antilog

$$5011.8 = \frac{E_o}{0.9}$$

$$4510.7 = E_o$$

Exponential Growth and Decay - Carbon Dating

$$A = A_0 2^{-t/k}$$

H) A bone originally contained 150 mg of carbon-14 now contains 85 mg of that isotope. Determine the approximate age of the bone to the nearest 100 years.

$$85 = (150)(2)^{-t/5570}$$

$$\frac{85}{150} = 2^{-t/5570}$$

$$\log\left(\frac{85}{150}\right) = \frac{-t}{5570} \log 2$$

$$-0.819 = -t/5570$$

$t = 4564$

≈ 4600 yrs

I) An isotope has a half-life of approximately 120 days. How many days will it take for a 12 mg sample of this isotope to decay to 7 mg? Round to the nearest day.

$$7 = (12)(2)^{-t/120}$$

$$\frac{7}{12} = 2^{-t/120}$$

$$\log\left(\frac{7}{12}\right) = \frac{-t}{120} \log 2$$

$$-0.2776 = -t/120$$

$t = 93.31$

≈ 93 days

J) A bone originally had 180 mg of carbon-14 now has 124 mg of carbon-14. How old is the bone?

$$124 = 180(2)^{-t/5570}$$

$$\frac{124}{180} = 2^{-t/5570}$$

$$\log\left(\frac{124}{180}\right) = \frac{-t}{5570} \log 2$$

$$-0.5377 = \frac{-t}{5570}$$

$t = 2994.75$

≈ 2995 yrs