Inverse Functions

Definition of Inverse Functions

a function that reverses another function

1. Determine if $f(x) = 6 - 2x$ and $g(x) = \dfrac{6-x}{2}$ are inverse functions.

Does $(f \circ g)(x) = x$?   Does $(g \circ f)(x) = x$? $(g \circ f)(x) =$

$\boxed{\text{yes}}$

$(f \circ g)(x)$ $6 - 2\left(\frac{6-x}{2}\right) = 6 - \dfrac{12 + 2x}{2} = \dfrac{12 - 12 + 2x}{2} = \dfrac{2x}{2} = x$ ✓

$\dfrac{6 - (6 - 2x)}{2} = \dfrac{6 - 6 + 2x}{2} = \dfrac{2x}{2}$
$= x$ ✓

2. Determine if $f(x) = 4 - x$ and $g(x) = x + 4$ are inverse functions.

$(f \circ g)(x) = 4 - (x+4) = 4 - x - 4 = -x$

$(g \circ f)(x) = 4 - x + 4 = -x + 8$ $\boxed{\text{no}}$

Property of Inverse Functions

If $f(x)$ and $f^{-1}(x)$ are inverse functions then $f(a) = b$ if and only if $f^{-1}(b) = a$.

How do you tell an equation is NOT a function?

If a vertical line passes through more than 1 point

Find the inverse of each function and determine whether or not the inverse is a function.

3. $f(x) = x + 3$ $f^{-1}(x) = x - 3$

① $y = x + 3$ $g(x)$
                    yes
② $x = y + 3$ can check by
③ $y = x - 3$ making sure
                    composition is identity

To find the inverse function

STEP 1: Rewrite the function with y =
        (if necessary)
STEP 2: Switch x and y in the equation
STEP 3: Solve for y
STEP 4: Replace y with the notation for an
        inverse function (if necessary)

$(f \circ g)(x) = x - 3 + 3 = x$
$(g \circ f)(x) = x + 3 - 3 = x$

4. $f(x) = x^2 - 4x + 4$

5. $g(x) = \dfrac{2}{3}x - \dfrac{1}{4}$

$y = \frac{2}{3}x - \frac{1}{4}$

$x = \frac{2}{3}y - \frac{1}{4}$

$y = \frac{3}{2}\left(x + \frac{1}{4}\right) = \frac{3}{2}x + \frac{3}{8}$

$\frac{2}{3}\left(\frac{3}{2}x + \frac{3}{8}\right) - \frac{1}{4} = x$ ✓

$\frac{3}{2}\left(\frac{2}{3}x - \frac{1}{4}\right) + \frac{1}{4} = x$ ✓ yes

Inverse Functions 8-8

6. $y = (x+10)^2$

7. $h(x) = \dfrac{x-7}{2}$

$$y = \frac{x-7}{2} \qquad x = \frac{y-7}{2}$$

$$2x = y-7$$

$$2x+7 = y$$

$$h^{-1}(x) = 2x+7$$

8. $q(x) = \dfrac{2}{3}x + 5$

$$y = \frac{2}{3}x + 5$$

$$x = \frac{2}{3}y + 5$$

$$x - 5 = \frac{2}{3}y$$

$$y = \frac{3}{2}(x-5) \text{ or } \frac{3}{2}x - \frac{15}{2}$$

9. $y = 7$

$$x = 7$$

Definition of Inverse Relations

If one relation contains (a,b) then the inverse must contain (b,a)

Find the inverse of each function and determine whether or not the inverse is a function.

10. {(5, 1), (1, 8), (-1, 4)}

$$\{ (1,5) \ (8,1) \ (4,-1) \}$$

yes

STEP 1: Switch x and y values

STEP 2: Look at new domain values to determine if it is a function

11. {(-5, 1), (2,- 8), (-3, 5), (0, 1)}

$$\{ (1,-5) \ (-8,2) \ (5,-3) \ (1,0) \}$$

not a function, x goes to 2 y values

12. {(6, 1), (3, - 7), (3, -4), (-8, 2)}

$$\{ (1,6) \ (-7,3) \ (-4,3) \ (2,-8) \}$$

yes