10-1 Exponential Functions

Exponential Functions

$y = x^2$ $\qquad$ $y = 2^x$

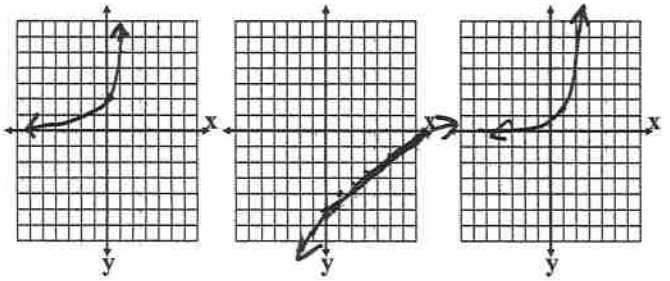
The function $f(x) = b^x$ is an **exponential function** with **base** b, where b is a positive real number other than 1 and x is any real number.

An **asymptote** is a line that a graph approaches (but does not reach) as its x- or y-values become very large or very small.

$y = ab^x$

Sketch the graph of each function. Then state the domain and range.

$y = 2(3)^x$ $\qquad$ $y = -5(4/5)^x$ $\qquad$ $y = 0.25(5)^x$



D = all reals
R = all positive #s

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For $y = ab^x$

When $a > 0$ and $b > 1$, the function $f(x) = b^x$ represents **exponential growth.**

When $a > 0$ and $0 < b < 1$, the function $f(x) = b^x$ represents **exponential decay.**

Determine whether each function represents exponential growth or decay.

a) $y = 4(2^x)$ $\qquad$ $a>0$ $b>1$ growth

b) $y = 0.3(1.2)^x$ $\qquad$ $a>0$ $b>1$ growth

c) $y = 6(2)^{-x}$ $\qquad$ $6(\frac{1}{2})^x$ decay $\qquad$ $a>0$ $b=\frac{1}{2}$

d) $y = 5(4/5)^x$ $\qquad$ $a>0$ $b<1$ decay

e) $y = 3(10)^{-x}$ $\qquad$ $3(\frac{1}{10})^x$ decay $\qquad$ $a>3$ $b=1/10$

f) $y = -5(4/5)^x$ $\qquad$ neither $a \neq 0$ $a<0$ neither

$x^2 \cdot x^3 = x^5$

$(x^2)^3 = x^6$

$16 = \frac{1}{b^2}$

$16b^2 = 1$

$b^2 = \frac{1}{16}$

$b = 1/4$

$b^0 = 1$

Write an exponential function whose graph passes through the given points.

- (0, -2) and (-2, -32)

$y = ab^x$ $\quad y = 2b^x$

$-2 = ab^0$ $\quad -32 = 2b^{-2}$

$-2 = a$

$16 = b^{-2}$

$\frac{1}{16} = b^2$

$1/4 = b$

- (0, 7) and (2, 63)

$y = ab^x$ $\quad y = 7b^x$

$7 = ab^0$ $\quad 63 = 7b^2$

$a = 7$ $\quad 9 = b^2$

$3 = b$

$y = (-2)(1/4)^x$

$y = 7(3)^x$

Simplify each expression.

A) $(3^{\sqrt{2}})^{\sqrt{2}}$ $\quad 3^{\sqrt{4}} = 3^2 = 9$

B) $25^{\sqrt{2}} \cdot 125^{\sqrt{2}}$ $\quad (5^2)^{\sqrt{2}} \cdot (5^3)^{\sqrt{2}} = 5^{5\sqrt{2}}$

C) $(x^{\sqrt{2}} y^{3\sqrt{2}})^{\sqrt{2}}$ $\quad x^{\sqrt{4}} y^{3\sqrt{4}} = x^2 y^6$

D) $(x^{\sqrt{6}})(x^{\sqrt{5}})$ $\quad x^{\sqrt{6}+\sqrt{5}}$

E) $(x^{\sqrt{6}})^{\sqrt{5}}$ $\quad x^{\sqrt{30}}$

F) $(2x^{\pi})(5x^{3\pi})$ $\quad 10x^{4\pi}$

Solve each equation or inequality.

A) $3^{n-2} = 27$

$3^{n-2} = 3^3$

$n - 2 = 3$

$n = 5$

B) $16^n < 8^{n+1}$

$(2^4)^n < (2^3)^{n+1}$

$4n < 3n + 3$

$n < 3$

C) $2^{2n} \geq 32$

$2^{2n} \geq 2^5$

$2n \geq 5$

$n \geq 2.5$

D) $32^{5p+2} \geq 16^{5p}$

$(2^5)^{5p+2} \geq (2^4)^{5p}$

$25p + 10 \geq 20p$

$5p + 10 \geq 0$

$5p \geq -10$

$p \geq -2$