Solving Quadratics by Completing the Square Notes

When completing the square, look at the problems below to determine a pattern.

A) $x^2 + 16x + 64$

$(x+8)^2$

B) $x^2 - 10x + 25$

$(x-5)^2$

C) $x^2 - 14x + 49$

$(x-7)^2$

Patterns you noticed: all perfect squares, end up with repeated factors, the value is half of b and the square root of c.

Perfect square trinomials:

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

- To find c – take the coefficient of the middle term, divide by 2 and square it
- To write a perfect square trinomial the sign depends on what the first sign is.
  - $(x \pm \frac{1}{2}$ of the second term's coefficient$)^2$

A) $x^2 + 4x + c$

$\frac{4}{2} = 2^2 = 4$

$c = 4$

$(x+2)^2$

B) $x^2 - 18x + c$

$\frac{-18}{2} = -9^2 = 81$

$c = 81$

$(x-9)^2$

C) $x^2 + 3x + c$

$\frac{3}{2} = 1.5^2 = 2.25$

$c = 2.25$

$(x+1.5)^2$

Steps for completing the square:

1) Make sure x's are on one side of the equation
2) Find c and add to each side of the equal sign
3) Simplify by writing as a perfect square trinomial.
4) Take the square root of each side.

   Always use ± when solving

A) $x^2 - 6x = 40$

$\frac{-6}{2} = -3^2 = 9$

$x^2 - 6x + 9 = 40 + 9$

$(x-3)^2 = 49$

$x - 3 = \pm 7$

$x - 3 = 7 \quad x - 3 = -7$

$\boxed{x = 10} \quad \boxed{x = -4}$

B) $x^2 - 4x - 5 = 0$

$\frac{-4}{2} = -2^2 = 4$

$x^2 - 4x + 4 = 5 + 4$

$(x-2)^2 = 9$

$x - 2 = \pm 3$

$x - 2 = 3 \quad x - 2 = -3$

$\boxed{x = 5} \quad \boxed{x = -1}$

C) $x^2 + 4x + 1 = 0$

$\frac{+4}{2} = 2^2 = 4$

$x^2 + 4x + 4 = -1 + 4$

$(x+2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$\boxed{x = -2 \pm \sqrt{3}}$

D) $x^2 + 12x + 4 = 0$

$\frac{12}{2} = 6^2 = 36$

$x^2 + 12x + 36 = 4 + 36$

$(x+6)^2 = 32$

$x + 6 = \pm\sqrt{32}$

$x + 6 = \pm 4\sqrt{2}$

$\boxed{x = -6 \pm 4\sqrt{2}}$

E) $x^2 + 6x + 15 = 0$

$\frac{6}{2} = 3^2 = 9$

$x^2 + 6x + 9 = -15 + 9$

$(x+3)^2 = -6$

$x + 3 = \pm\sqrt{-6}$

$x + 3 = \pm i\sqrt{6}$

$\boxed{x = -3 \pm i\sqrt{6}}$

F) $x^2 + 11x + 24 = 0$

$\frac{11}{2} = 5.5^2 = 30.25$

$x^2 + 11x + 30.25 = -24 + 30.25$

$(x+5.5)^2 = 6.25$

$x + 5.5 = \pm 2.5$

$\boxed{x = -3} \quad \boxed{x = -8}$

G) $x^2 - x = -4$

$\frac{-1}{2} = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$x^2 - x + \frac{1}{4} = -4 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = -15/4$

$x - \frac{1}{2} = \pm i\sqrt{\frac{15}{4}}$

$\boxed{x = \frac{1}{2} \pm \frac{i\sqrt{15}}{2}}$

or $\frac{1 \pm i\sqrt{15}}{2}$