Combinations Notes

In a **combination**, order does **NOT** matter.

If calculating how to arrange 3 people in a class of 30 and the arrangement matters, this would be a permutation. If calculating how many different groups of 3 could exist out of a class of 30, then this would be a combination. The order doesn't matter, just who is in each group does.

Formula for calculating combinations:

Choosing from "n" elements "r" at a time:

$$\frac{n!}{r!(n-r)!}$$ or nCr using the calculator.

Remember n is always larger than r

To use the calculator for nCr: Enter the first number (n) → Math → PRB → 3 → then the last number (r).

Evaluate the following by hand and check with the calculator.

${}_6C_4 = \frac{6!}{4!(6-4)!}$ ${}_{11}C_3 = 165$ ${}_{12}C_{10} = 66$

$\frac{6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times2\times1} = \frac{6\times5}{2\times1} = 15$ $\frac{11\times10\times9}{3\times2\times1}$

A) How many 3-element subsets can be formed from the set {a, b, c, d, e,}? $5C3 = 10$

B) Suppose your teacher selects 3 students from a class of 21 for a reward. How many combinations are possible?

$21C3 = 1330$

C) Four tickets to a concert are to be given away. There are 20 people who have entered the contest for tickets. How many different combinations of winners are possible? $\frac{20!}{4!\,16!}$ $20C4 = 4845$

D) How many different groups did not receive tickets from part C? What is the relationship between these two answers? Why is this the case?

$20C16 = 4845$ $\frac{20!}{16!\,4!}$

E) Bob is hanging a hammock in his back yard. Two trees are required to hang the hammock. Bob can select any of the eight trees in his yard and no three are collinear. How many different ways can Bob hang his hammock? $8C2 = 28$

F) How many diagonals can be drawn from the vertices of a pentagon?

$5C2 = 10 - 5 = 5$

G) How many diagonals can be drawn from the vertices of a hexagon? $6C2 = 15 - 6 = 9$

H) How many 3-element subsets can be found in {1, 2, 3, 4, 5, 6}?

I) There are 8 men and 12 women that can be selected to serve on a committee of three. How many different committees are possible if the committee is all men? All women?

$8C3 = 56$ $12C3 = 220$

J) From question I, how many different committees could be formed if there were 6 selected to serve on the committee – 3 men and 3 women?

$56 \cdot 220 = 12,320$

K) How many different card hands can be chosen from a deck of cards having exactly 3 kings and 4 aces? $4C3 = 4$ $4C4 = 1$ $4 \times 1 = 4$

L) How many different 7 card hands can be formed with 3 black cards and 4 red cards? $26C3 = 2600$ $26C4 = 14,950$ $38,870,000$

M) How many lines can be drawn from 4 points if no three are collinear? $4C2 = 6$

N) A wallet has one of each - $1, $5, $10, and $20 bills. How many different sums of change are possible? *keep in mind that we could use 1 bill, 2 bills, 3 bills, or 4 bills to get a different amount of money.

Select 1: $4C1 = 4$ $1, $5, $10, or $20

Select 2: $4C2 = 6$ $1 & $5, $1 & $10, $1 & $20, $5 & $10, $5 & 20, $10 & 20

Select 3: $4C3 = 4$ (1, 5, 10) (1, 5, 20) (1, 10, 20) (5, 10, 20)

Select 4: $4C4 = 1$ (1, 5, 10, 20)

15 options