

# Arithmetic Series

## Arithmetic Series

- ▶  $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$
- ▶ Series: the sum of the terms in a sequence.
- ▶ Arithmetic Series: the sum of the terms in an arithmetic sequence.
- ▶ Arithmetic Sequence:  
2, 4, 6, 8, 10
- ▶ Corresponding arithmetic series:  
 $2 + 4 + 6 + 8 + 10$

## Arithmetic Series

- ▶  $S_n$  is the symbol used to represent the first 'n' terms of a series.
- ▶ Given the sequence 1, 11, 21, 31, 41, 51, 61, 71, ... find  $S_4$
- ▶ We add the first four terms

$$1 + 11 + 21 + 31 = 64 = S_4$$

## Arithmetic Series Formula

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1$  = first term

$a_n$  = the nth term

$n$  = # of terms

Find the sum of the first 10 terms of the arithmetic series with  $a_1 = 6$  and  $a_{10} = 51$

$$S_{10} = \frac{10}{2}(6 + 51) = 285$$

Find  $S_n$  for each arithmetic series described.

1. Find the sum of the first 6 terms if:

$$a_1 = 3 \text{ and } a_6 = 20$$

$$n=6$$

$$S_6 = \frac{6}{2}(3+20)$$

$$S_6 = 69$$

2. Find the sum of the first 30 terms if:

$$a_1 = 15 \text{ and } a_{30} = -12$$

$$S_{30} = \frac{30}{2}(15 - 12)$$

$$S_{30} = 45$$

Find  $S_n$  for each arithmetic series described.

3.  $a_1 = 7, a_n = 79, n = 8$

$$S_8 = \frac{8}{2}(7+79)$$

$$S_8 = 344$$

4.  $a_1 = 7, d = -2, n = 9$

$$S_9 = \frac{9}{2}(7-9)$$

$$= -9$$

$$a_n = a_1 + d(n-1)$$

$$a_9 = 7 + -2(9-1)$$

$$a_9 = -9$$

5.  $a_1 = 4, d = -5, n = 11$

## Examples

Last value of n

Greek Symbol Sigma

Formula used to find the sequence

$$\sum_{n=1}^5 (n+1)$$

First value of n

2, 3, 4, 5, 6

This means to find the sum of the series  $n+1$ , where we plug in the values 1 - 5 for  $n$ .

1<sup>st</sup>:

$$1+1 = a_1 = 2$$

$$2+1 = 3 = a_2$$

$$3+1 = 4 = a_3$$

$$4+1 = 5 = a_4$$

$$5+1 = 6 = a_5$$

$$2+3+4+5+6 = 20$$

$$S_5 = \frac{5}{2}(2+6)$$

$$= 20$$

Find the sum of each arithmetic series.

$$\sum_{n=1}^6 (2n+11)$$

$$a_1 = 2(1) + 11 = 13$$

$$a_6 = 2(6) + 11 = 23$$

$$S_6 = \frac{6}{2}(13+23)$$

$$S_6 = 108$$

Find the sum of each arithmetic series.

$$\sum_{n=7}^{11} (42 - 9n)$$

$$\sum_{n=1}^6$$

$$n-1+1$$

$$n=6-1+1=6$$

$$11-7+1=5$$

$$a_7 = 42 - 9(7) = -21$$

$$a_{11} = 42 - 9(11) = -57$$

$$S = \frac{5}{2}(-21 - 57)$$

$$= -195$$

Express the related arithmetic series in Sigma Notation. Find the sum of the finite series.

9.  $a_1, a_2, a_3, a_4, a_5$   
6, 14, 22, 30, 38

$$\sum_{n=1}^5 8n - 2$$

$$6 + 8(n-1)$$

$$6 + 8n - 8$$

$$-2 + 8n$$

$$\text{or } 8n - 2$$

$$S_5 = \frac{5}{2}(6 + 38)$$

$$S_5 = 110$$

10. 12, 19, 26, 33, 40, 47, 54

$$\sum_{n=1}^7 7n + 5$$

$$12 + 7(n-1)$$

$$12 + 7n - 7$$

$$7n + 5$$

$$S_7 = \frac{7}{2}(12 + 54)$$

$$S_7 = 231$$