

ARITHMETIC SEQUENCES

SOMETHING TO THINK ABOUT...

- Every day a radio station asks a question for a prize of \$150. If the 5th caller does not answer correctly, the prize money increased by \$150 each day until someone correctly answers their question.
- At your table:
 - Make a list of the prize amounts for a week (Mon - Fri) if the contest starts on Monday and no one answers correctly all week.

PRIZE AMOUNTS

- Monday : 150
- Tuesday: 300
- Wednesday: 450
- Thursday: 600
- Friday: 750

ARITHMETIC SEQUENCES

- These prize amounts form a sequence, more specifically each amount is a term in an arithmetic sequence.
- How do we find the next term?

adding 150

KEY TERMS TO KNOW:

- **Sequence:** a list of numbers in a specific order.
- **Term:** each number in a sequence
- **Arithmetic Sequence:** a sequence in which each term after the first term is found by adding a constant, called the common difference (d), to the previous term.

DEVELOPING A FORMULA

- What if I wanted to find the 50th (a_{50}) term of the sequence 2, 5, 8, 11, 14, ...?
- What is the common difference?
+3
- What is the relationship between the term we are finding and the number of times the common difference, d , is added?

one less than
the term number

- The formula for finding any term in an arithmetic sequence is

$$a_n = a_1 + d(n-1).$$

- ***Note: it IS POSSIBLE for common differences to be negative.

EXAMPLE:

- Let's go back to our first example about the radio contest.
Suppose no one correctly answered the question for 15 days.
What would the prize be on day 16?

$$a_{16} = 150 + 150(16-1)$$
$$a_{16} = \$2400$$

$$a_1 = 150$$
$$d = 150$$
$$n = 16$$
$$a_{16}$$

ARITHMETIC SEQUENCES EXAMPLES

• Find the next three terms of the following sequences:

1) 6, 11, 16, 21, ... $d=5$

26, 31, 36

2) 200, 185, 170, 155, ... $d=-15$

140, 125, 110

3) $4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \dots$

• Find the first 5 terms of the sequence described:

4) $a_1 = 12, d = -8$

12, 4, -4, -12, -20

5) $a_1 = -6, d = 10$

-6, 6, 16, 26, 36

6) $a_1 = 3, d = -4$

3, -1, -5, -9, -13

FIND THE INDICATED TERM OF EACH ARITHMETIC SEQUENCE.

$$a_n = a_1 + d(n-1)$$

7. $a_1 = 3, d = 7, n = 14$

$$a_{14} = 3 + 7(14-1)$$

$$a_{14} = \boxed{94}$$

8. $a_1 = -4, d = -9, n = 20$

$$a_{20} = -4 - 9(20-1)$$

$$a_{20} = \boxed{-175}$$

9. $a_1 = 5/2, d = -3/2, n = 11$

$$a_{11} = 5/2 - 3/2(11-1)$$

$$a_{11} = 5/2 - 15 = \boxed{-\frac{25}{2} = -12.5}$$

WRITE AN EQUATION FOR THE NTH TERM OF EACH ARITHMETIC SEQUENCE:

10. 7, 16, 25, 34, ...

$$a_1 = 7$$

$$d = 9$$

$$a_n = 7 + 9(n-1)$$

$$a_n = 7 + 9n - 9$$

11. 18, 11, 4, -3, ...

$$a_1 = 18$$

$$d = -7$$

$$a_n = 18 - 7(n-1) = 18 - 7n + 7$$

$$a_n = 9n - 2$$

12. -3, -5, -7, -9, ...

$$a_n = -3 - 2(n-1)$$

$$a_n = -3 - 2n + 2$$

$$\Rightarrow a_n = -2n - 1$$

$$a_n = -7n + 25$$

ARITHMETIC MEANS

Arithmetic Means: the terms between any two nonconsecutive terms of an arithmetic sequence.

ex. 17, 10, 3, -4, -11, -18, ...

• Between 10 and -18 there are three arithmetic means 3, -4, -11.

Find three arithmetic means between 8 and 14.

$$8, 9.5, 11, 12.5, 14$$

$$a_5 = a_1 + d(5-1)$$

$$14 = 8 + d(4)$$

$$6 = 4d \quad d = \frac{3}{2} \text{ or } 1.5$$

FIND THE ARITHMETIC MEANS IN EACH SEQUENCE

13. 55, 70, 85, 100, 115

$$115 = 55 + d(5-1)$$

$$60 = 4d$$

$$d = 15$$

14. -8, -5, -2, 1, 4, 7

$$a_6 = -8 + d(6-1)$$

$$7 = -8 + 5d$$

$$15 = 5d \quad d = 3$$

15. 10, 4, -2, -8

$$-8 = 10 + d(4-1)$$

$$-18 = 3d$$

$$-6 = d$$

EXAMPLE

$$n = ?$$

$$d = 7$$

• 72 is the 12th term of the sequence -5, 2, 9, ...

$$a_1 = -5$$

$$a_n = 72$$

$$72 = -5 + 7(n-1)$$

$$72 = -5 + 7n - 7$$

$$72 = -12 + 7n$$

$$84 = 7n \quad n = 12$$

COMPLETE THE STATEMENT FOR EACH ARITHMETIC SEQUENCE

16. 170 is the 30th term of -4, 2, 8, ...

$$170 = -4 + 6(n-1)$$

$$180 = 6n$$

$$170 = -4 + 6n - 6$$

$$n = 30$$

$$170 = -10 + 6n$$

17. 124 is the 19th term of -2, 5, 12, ...

$$124 = -2 + 7(n-1)$$

$$133 = 7n$$

$$124 = -2 + 7n - 7$$

$$19 = n$$

$$124 = -9 + 7n$$