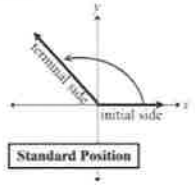
Angles and Angle Measure

Angle Measurement: Terms to Know

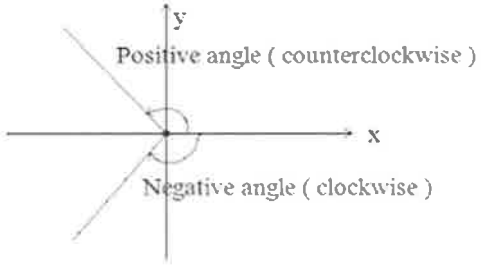
Initial Side- fixed along the positive x-axis

Terminal Side- other ray of the angle, rotate about the center

Standard Position- vertex at the origin, initial side along the positive x-axis



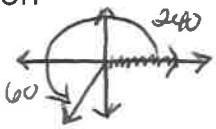
The measure of an angle is determined by the amount and direction of rotation from the initial side to the terminal side.



Draw an angle in standard position

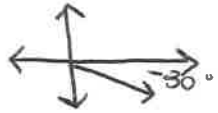
- $240° = 180 + 60$

60° CCW past negative x-axis



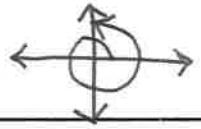
- $-30°$

30° CW from positive x-axis



- $450° = 360° + 90°$

90° CCW past positive x-axis



Radians

- Another unit used to measure angles is a radian. The definition of a radian is based on the **unit circle.**
- **Unit Circle-** a circle of radius 1 unit whose center is at the origin of a coordinate system.
- One radian is the measure of an angle Θ in standard position whose rays intercept an arc of length 1 unit on the unit circle.

Radian and Degree Conversions

- Circumference of any circle: $2\pi r$
- Circumference of the unit circle: 2π, so 2π radians = 360°
- To rewrite the radian measure of an angle in degrees, multiply the number of radians by: $\frac{180}{\pi \ radians}$
- To rewrite the degree measure of an angle in radians, multiply the number of degrees by: $\frac{\pi \ radians}{180}$

Example: Rewrite the degree measure in radians and the radian measure in degrees.

60°

$60^\circ = 60^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$\frac{60\pi}{180} = \frac{\pi}{3}$

$\frac{-7\pi}{4}$

$-\frac{7\pi}{4} = -\frac{7\pi}{4}\left(\frac{180}{\pi}\right)$

$= \frac{-1260}{4}$ or -315°

Example 3: Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 1:00 pm to 3:00 pm.

Clock divided into 12 = parts

1 to 3 = $\frac{2}{12} = \frac{1}{6}$

$\frac{1}{6}$ of $360 = 60^\circ$

CW: $-60^\circ = -\pi/3$ radians

Coterminal Angles: Two angles in standard position that have the same terminal sides

For example, a 405° angle and a 45° angle

Notice: 405-45=360. In degree measure, coterminal angles always differ by an integral multiple of 360.

You can find an angle that is coterminal to a given angle by adding or subtracting a multiple of 360. In radian measure, this would mean adding or subtracting a multiple of 2∏.

Example 4: Find one angle with positive measure and one angle with negative measure that is coterminal with each angle.

240

$+: \quad 240 + 360 = 600^\circ$

$-: \quad 240 - 360 = -120^\circ$

$\frac{9\pi}{4}$

$+: \quad \frac{9\pi}{4} + 2\pi = \frac{17\pi}{4}$

$-: \quad \frac{9\pi}{4} - 2(2\pi)$

$= -\frac{7\pi}{4}$