Lesson 10-4 - Inscribed Angles

Inscribed Angle: An angle whose vertex lies on a circle and whose sides are chords of the circle (or one side tangent to the circle).

$\angle ABC$ is an inscribed angle.

Examples:

Yes!

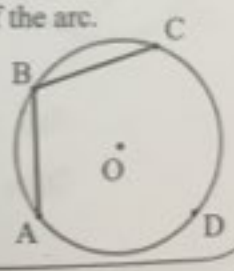
Yes!

Intercepted Arc

Intercepted Arc: An angle intercepts an arc if and only if each of the following conditions holds:

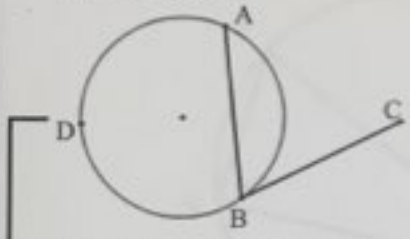
1. The endpoints of the arc lie on the angle.
2. All points of the arc, except the endpoints, are in the interior of the angle.
3. Each side of the angle contains an endpoint of the arc.

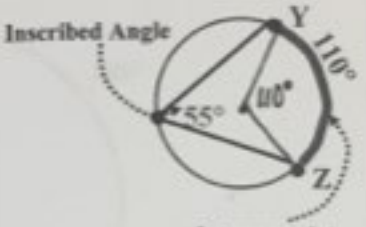
$\overset{\frown}{ADC}$ *is the* int*ercepted arc of* $\angle ABC$.



Inscribed Angle Theorem

The measure of an inscribed angle is equal to ½ the measure of the intercepted arc.

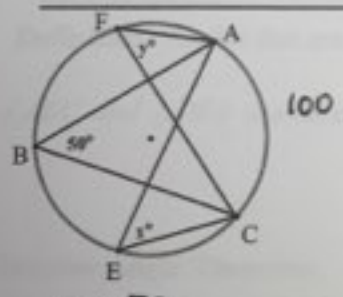




An angle formed by a chord and a tangent can be considered an inscribed angle.

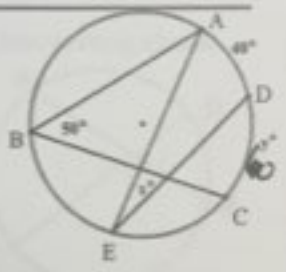
$$m\angle ABC = \frac{m\overset{\frown}{AB}}{2}$$

Examples: Find the value of x and y in the fig.



X = 50

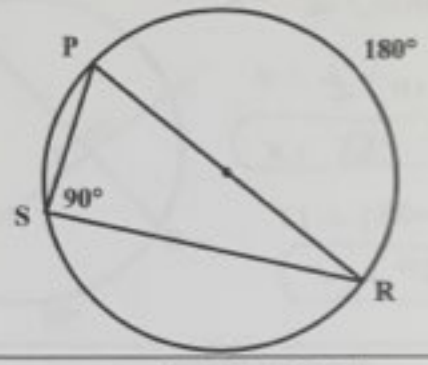
y = 50



X = 20

y = 60

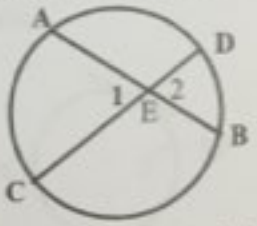
An angle inscribed in a semicircle is a right angle.



Interior Angle Theorem

Definition: Angles that are formed by two intersecting chords.

∠AEC and ∠DEB are int*erior angles.*

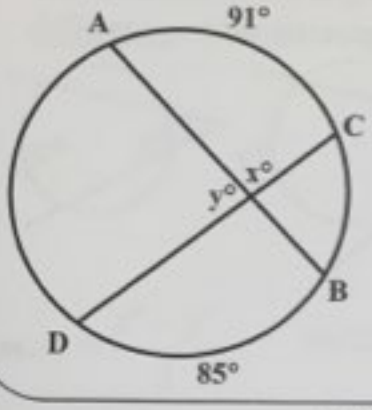


Interior Angle Theorem:

The measure of the angle formed by the two intersecting chords is equal to ½ the sum of the measures of the intercepted arcs.

$$m\angle 1 = m\angle 2 = \frac{m\overset{\frown}{AC} + m\overset{\frown}{DB}}{2}$$

Example: Interior Angle Theorem



$x = \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{DB})$

$x = \frac{1}{2}(91 + 85)$

$x = 88°$

$y = 180 - 88$

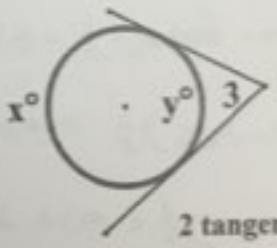
$y = 92°$

Exterior Angles

An angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside the circle.

Two secants

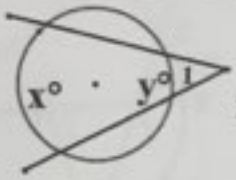
A secant and a tangent



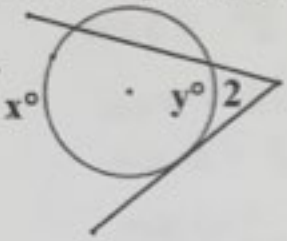
2 tangents

Exterior Angle Theorem

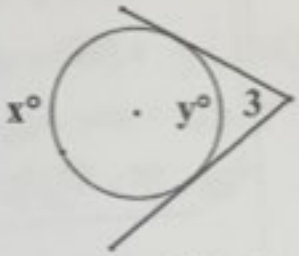
The measure of the angle formed is equal to ½ the difference of the intercepted arcs.



$$m\angle 1 = \frac{x^\circ - y^\circ}{2}$$



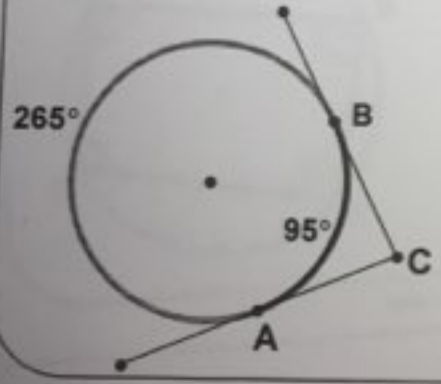
$$m\angle 2 = \frac{x^\circ - y^\circ}{2}$$



$$m\angle 3 = \frac{x^\circ - y^\circ}{2}$$

Example: Exterior Angle Theorem

In the given figure find the $m\angle ACB$.



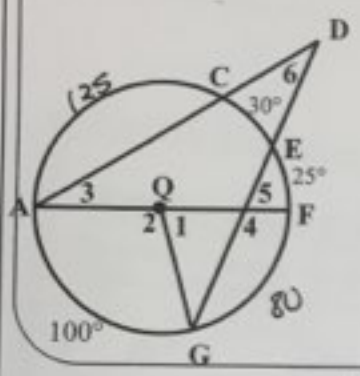
$m\angle ACB = \frac{1}{2}(m\overset{\frown}{ADB} - m\overset{\frown}{AB})$

$m\angle ACB = \frac{1}{2}(265 - 95)$

$m\angle ACB = \frac{1}{2}(170)$

$m\angle ACB = 85^\circ$

Given $\overline{AF}$ is a diameter, $m\overset{\frown}{AG} = 100°$, $m\overset{\frown}{CE} = 30°$ and $m\overset{\frown}{EF} = 25°$.
Find the measure of all numbered angles.



$m\angle 1 = 80$

$m\angle 2 = 100$

$m\angle 3 = \frac{55}{2} = 27.5°$

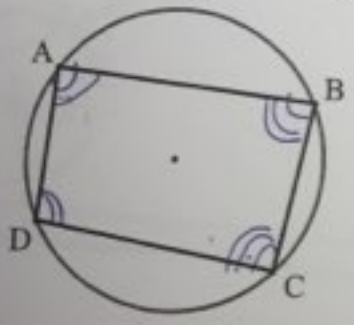
$m\angle 4 = \frac{1}{2}(80 + 155) = \frac{1}{2}(235) = 117.5°$

$m\angle 5 = \frac{1}{2}(25 + 100) = 62.5°$

$m\angle 6 = \frac{100 - 30}{2} = \frac{70}{2} = 35°$

Inscribed Quadrilaterals

If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.



$m\angle DAB + m\angle DCB = 180°$

$m\angle ADC + m\angle ABC = 180°$