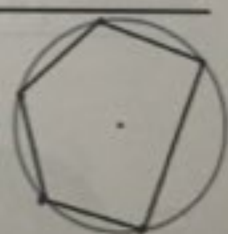
Lesson 10-3

Arcs and Chords

Polygons

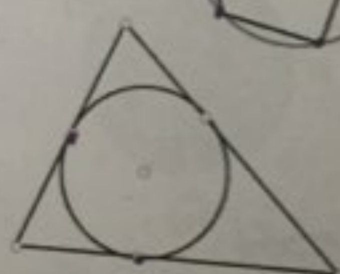
Inscribed Polygon:

A polygon inside the circle whose vertices lie **on the circle.**



Circumscribed Polygon :

A polygon whose sides are **tangent to a circle.**



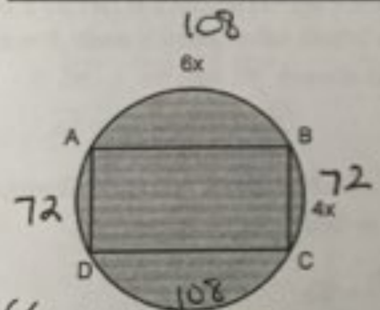
Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.

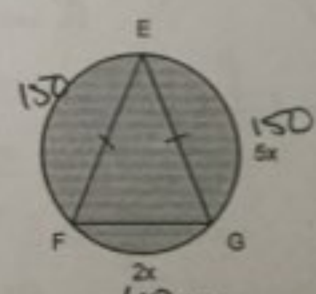
- Triangle $\frac{360}{3} = 120°$

- Square $\frac{360}{4} = 90°$

- 20-gon $\frac{360}{20} = 18°$

Determine the measure of each arc of the circle circumscribed about the polygon.





$2((6x+4x) = 360$

$2(10x) = 360$

$20x = 360$

$x = 18$

$12x = 360$

$x = 30$

$\overset{\frown}{AB} = \overset{\frown}{DC} = 108$

$\overset{\frown}{BC} = \overset{\frown}{AD} = 72$

$\overset{\frown}{EF} = \overset{\frown}{EG} = 150$

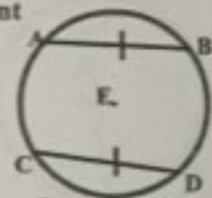
$\overset{\frown}{FG} = 60$

Theorem #1:

In a circle, if two chords are congruent then their corresponding minor arcs are congruent and vice versa.

If $AB = CD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $AB = CD$



Example: Given $m\overset{\frown}{AB} = 127^\circ$, find the $m\overset{\frown}{CD}$

Since $m\overset{\frown}{AB} = m\overset{\frown}{CD}$

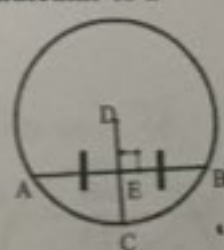
$m\overset{\frown}{CD} = 127^\circ$

Theorem #2:

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

If $\overline{DC} \perp \overline{AB}$ *then* $\overline{DC}$ *bisects* $\overline{AB}$ *and* $\overset{\frown}{AB}$

$\therefore \overline{AE} \cong \overline{BE}$ *and* $\overset{\frown}{AC} \cong \overset{\frown}{BC}$



Example: If AB = 5 cm, find AE.

If $m\ \overset{\frown}{AB} = 120^\circ$, *find* $m\ \overset{\frown}{AC}$

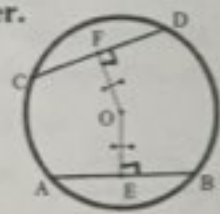
$$AE = \frac{AB}{2} \quad \therefore AE = \frac{5}{2} = 2.5\ cm$$

$$m\overset{\frown}{AC} = \frac{m\overset{\frown}{AB}}{2}, \therefore m\overset{\frown}{AC} = \frac{120}{2} = 60^\circ$$

Theorem #3:

In a circle, two chords are congruent if and only if they are equidistant from the center.

$\overline{CD} \cong \overline{AB}$ *iff* $\overline{OF} \cong \overline{OE}$



Example: If AB = 5 cm, find CD.

Since AB = CD, CD = 5 cm.

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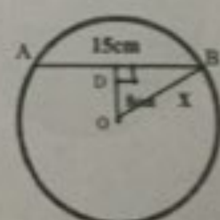
Try Some Sketches:

- Draw a circle with a chord that is 15 inches long and 8 inches from the center of the circle.
- Draw a radius so that it forms a right triangle.
- How could you find the length of the radius?

Solution: ΔODB is a right triangle and $\overline{OD}$ *bisects* $\overline{AB}$

$$DB = \frac{AB}{2} = \frac{15}{2} = 7.5 \text{ cm}$$
$$OD = 8 \text{ cm}$$
$$OB^2 = OD^2 + DB^2$$
$$OB^2 = 8^2 + (7.5)^2 = 64 + 56.25 = 120.25$$
$$OB = \sqrt{120.25} \approx 11cm$$



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$7.5^2 + 8^2 = c^2$

$56.25 + 64 = c^2$

$120.25 = c^2$

$\sqrt{120.25} = c$

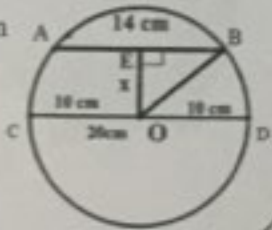
$10.97 \approx c$

Try Some Sketches:

- Draw a circle with a diameter that is 20 cm long.
- Draw another chord (parallel to the diameter) that is 14cm long.
- Find the distance from the smaller chord to the center of the circle.

Solution: $\overline{OE}$ *bisects* $\overline{AB}$. $\therefore EB = \dfrac{AB}{2} = \dfrac{14}{2} = 7cm$

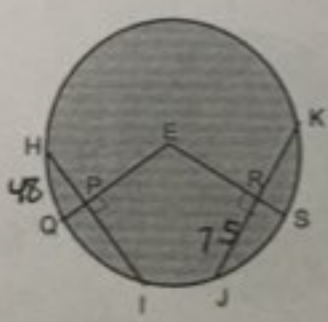
ΔEOB is a right triangle. OB (radius) = 10 cm



$$OB^2 = OE^2 + EB^2$$
$$10^2 = X^2 + 7^2$$
$$X^2 = 100 - 49 = 51$$
$$X = \sqrt{51} = \textbf{7.1 cm}$$

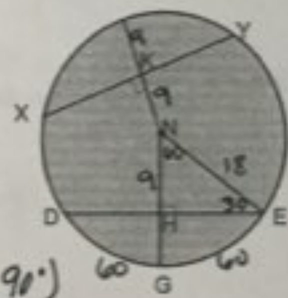
In circle E, m$\widehat{HQ}$ = 48°, HI = JK, and JR = 7.5. Find each measure.

- m$\widehat{HI}$ = 96
- m$\widehat{QI}$ = 48
- m$\widehat{JK}$ = 96
- HI = 15
- PI = 7.5
- JK = 15



The radius of circle N is 18, NK = 9, $m\widehat{DE} = 120$. Find each measure.

- $m\widehat{GE}$ = 60
- m∠HNE = 60
- m∠HEN = 30
- HN = 9 (use 30°, 60°, 90°)



The radius of circle O = 32, $\widehat{PQ} = \widehat{RS}$, and PQ = 56. Find each measure.

- PB = 28
- BQ = 28
- OB $28^2 + OB^2 = 32^2$
  $OB^2 = 240$
  $OB = 15.49$
- RS = 56